**Math 201** **Quiz II: Practice List** *N. Nahlus (2014)*

**1)** Let $f(x,y) = \sqrt{9 - x^2 - y^2}\ +5$

a) Find the domain D of $f$ & is D open, closed, bounded ?
b) Describe the level curves of $f$ & what is the level curve of $f$ passing through (1, 2)
c) Sketch the graph of z = $\sqrt{9 - x^2 - y^2}\ +k$ for $k$ =0, 5.
d) From definitions, show that $f_x$ (0, 0) = 0 = $f_y$ (0, 0)
e) From definitions, prove that $f$ is differentiable at (0, 0).

**1*)** See the syllabus problems on 14.1

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**2)** Let $g(x,y) = \sqrt{9 - x^2 - y^2}$ +5x +5y +1
i) From definitions, show that $f_x$ (0, 0) = 5 = $f_y$ (0, 0)
ii) From definitions, prove that $f$ is differentiable at (0, 0)

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**3)** (a) Show that $\lim\limits_{(x,y\to(0,0)} \dfrac{xy^2e^{2y}}{x^6 + 3y^2e^{2y}} \cos(\dfrac{y}{x}) = 0$ (b) Investigate $\lim\limits_{(x,y\to(0,1)} \dfrac{x(y-1)^4}{x^2 + (y-1)^8}$

(c) Let $f(x, y) = 4\cos(\dfrac{xy^5}{x^2 + y^8}) + 2xy + 1$ for $(x, y) \neq (0,0)$

*Prove or disprove* that $f(0,0)$ can be defined so that $f(x, y)$ is continous at (0, 0).

3*) *Let f(x,y)=0 if x =0 or y=0 and f(x,y)=1 otherwise.*
*i) Investigate lim f(x,y) as (x,y) goes (0,0).*
*ii) Investigate lim g(x,y)= x^2/(|x|+|y|) as (x,y) goes (0,0).*

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4) Suppose $F(x,y,z,w) = 100$ and all components of $\nabla F$ are never zero.

Find $\dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial y} \cdot \dfrac{\partial y}{\partial x}$ given that $\dfrac{\partial x}{\partial z} = e^{3x-10y+7z}$. Justify your answer .

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5) The function $f(x, y, z)$ at a point p increases most rapidly in the direction of the vector v=(3,4,5) with directional derivative $10\sqrt{2}$ .

(i) Find $\nabla f(P)$

(ii) Find the directional derivative of $f(x, y, z)$ at $P$ in the direction of the vector w=(4, 0, 3).

(iii) Is it possible to find a vector v such that $D_v$ (f) (P) = 20 ? Explain.

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5*) The function $f(x, y, z)$ at a point p decreases most rapidly in the direction of v=(3,4,5) with directional derivative $-10\sqrt{2}$ .

(i) Find $\nabla f(P)$ (ii) Find the directional derivative of $f(x, y, z)$ at $P$ in the direction of the vector w=(4, 0, 3).

(iii) Is it possible to find a vector v such that $D_v$ (f) (P) = 15 ? (Hint: $\sqrt{2}$ = 1.4.14..). Explain.

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5**) The derivative of of $f(x; y)$ at $P(1; 2)$ in the direction of i + j is $2\sqrt{2}$
and in direction of –2j is –3. Find the derivative of $f$ in the direction of –i–2j.
(Extra Hint: Suppose grad(f)(P)= <a, b>. So you have 2 equations in 2 unknowns

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0) Find a, b if $f(x, y, z) = e^{ax+by}\cos 5z$ satisfies Laplace equation $f_{xx} + f_{yy} + f_{zz} = 0$ .

0**) Prove that $f(x,y,z) = (x^2 + y^2 + z^2)^{1/2}$ satisfies Laplace equation $f_{xx} + f_{yy} + f_{zz} = 0$ .

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6) Find the parametric equations for the line tangent to the curve of intersection of the surfaces $xyz = 1$ and $x^2 + y^2 + 3z^2 = 5$ at the point $P(1; 1; 1)$. (big Hint: use cross products).

Baby 7) Given the surface $z - x^2 + 4xy = y^3 + 4y - 2$ containing the point $P(1; -1; -2)$

a) Find an equation of the tangent plane to the surface at $P$.

b) Find an equation of the normal line to the surface at $P$.

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8) By about how much will $f(x; y; z) = \ln\sqrt{x^2 + y^2 + z^2}$ change if the point $p(x; y; z)$ moves from $P_0(3; 4; 12)$ a distance of 0.1 units in the direction of 3i + 6j − 2k ? (see Thomas p. 794)

8*) By about how much will $f(x; y; z) = \ln\sqrt{x^2 + y^2 + z^2}$ change if the point $p(x; y; z)$ moves from $P_0(3; 4; 12)$ to $P_1(3.01; 4.03; 12.01)$.

*Extra Hint*: You may use the "sister" formula: $\Delta f \sim f_x\,(P_0)\,\Delta x + \cdots \ldots \Delta y + \cdots \ldots \Delta z$

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9) (14.4: Chain Rule) Suppose $\nabla f(1,1,1) = \mathbf{5i} + 3\mathbf{j} + 4\mathbf{k}$ and $f(1,1,1)=5$.

Let $P = f(t^4, t^2, tx^2)$ where $f(u,v,w)$ is a differentiable function. Then at t=1, x=1,

$(i)$ $\partial P/\partial t = \ldots\ldots$ $(ii)$ $\partial(x^3P)/\partial t = \ldots.$ $(iii)$ $\partial(t^3P)/\partial t = \ldots\ldots$

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10) (14.4: Chain Rule) Suppose $f(tx, ty) = t^5 f(x,y)$ for all values of $x, y, t$ (where $f(u,v)$ is a differentiable function).

Show that $(i)$ $xf_x + yf_y = 5f$ ( <u>Hint</u>: Partial w.r.t t both sides, then set t=1).

$(ii)$ $x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy} = 20f$ (Hint: Double partial w.r.t t both sides, then set t=1).

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<u>11</u>) Find the set of points <u>on the surface</u> $x^2 + y^2 - 36 = 8xyz$ where the tangent plane is

(i) perpendicular to the x-y plane. (ii) parallel to the x-y plane.

<u>12</u>) TBA on 12.6: Match the surfaces ..…………………………

13-113) See ALL Lecture & Recitation Problems

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<u>Section 14.7 A</u>:

114a) Investigate the critical points of

$f(x,y) = 2x^3 + 6xy + 2y^3 + 17$ for local maxima, local minima, or saddle points.

114b) Locate all local extrema and saddle points of $f(x,y) = x^3 - y^3 - 2xy + 6$

114c) Locate all local extrema and saddle points of $f(x,y) = 4xy - x^4 - y^4$.

*Good Luck*